ABSTRACT:

Optical flow as an image processing technique finds its applications in many areas of computer vision. A lot of research has been carried out in order to improve optical flow detection techniques, but this field is not perfect even as of today. Many methods have been proposed to solve the optical flow problem however, instabilities at the boundaries of moving objects are still challenges. There are many methods to extract optical flow, yet there is no platform that brings out comparison on the performance of these methods. In this paper, the two major optical flow algorithms are stated. Also I propose two methods of coarse to fine estimation to improve results of basic approach.

Keywords: Optical flow, Gradient constraint equation, Horn-Schunck Algorithm, Lucas-Kanade, Global smoothness, Coarse to fine, Brightness Constancy, Aperture Problem, Pyramid

[1] INTRODUCTION

[1] Optical flow is defined as the apparent motion of image brightness patterns in an image sequence. There are many methods to extract optical flow out of which, the gradient method is the basic method. But Gradient method cannot give a complete solution for optical flow fields because of the aperture problem. Hence to obtain a complete solution, two different differential techniques namely Horn-Schunck algorithm and Lucas-Kanade algorithm are analyzed and are combined with coarse to fine estimation for better results. I propose two methods of coarse to fine.

By optical flow we mean apparent movement of the pixels on the image plane. As it can be understood, if we take several images with the same camera at different times, objects might move in the image due to the fact that either a) the objects seen in the image move or b) the camera moves (ego motion). Based on the Lambertian surface model, we expect that any change in the illumination level of a pixel is due to movement, as seen in the following figure.

Figure: 1. Movement of pixels from t to t+1

(a) \( I(x, y, t) \)  
(b) \( I(x+v, y+v, t+1) \)
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The [Figure-1] above shows two different images taken at times t and t+1. Movement of the pixel is registered as a change in the illumination level of the pixel. As it can be understood, there exists several different ways of finding the corresponding pixels, not necessarily based on the ‘illumination’ (gray) level, but other more robust image features. The one that I explain below is a so called ‘variational’ model (based on the calculus of variations), based on the original model of Horn & Schunck.[1]

From the above we come up with the following constancy assumption:
I (x, yt) = I (x+u, y+v, t+1)

All optical flow methods are based on the following assumptions:

1) Color constancy (brightness constancy for single-channel images).
2) Small motion.

With these assumptions, if we have two images (say two adjacent frames of a video), what we need to do is simply find pixel correspondences between the two images. Because of color constancy, we don’t need to consider the RGB value’s change between two images, and because of small motion, we can find the corresponding point of a pixel within a very little neighborhood. The basic method that has been developed to extract optical flow is the gradient method.

Assume brightness constancy:

\[ f(x,y,t) = f(x + dx, y + dy, t + dt) \]

Assume small motion (first order Taylor expansion):

\[ f(x,y,t) = f(x,y,t) + \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial t}dt \]

Then we can get the Constraint Equation of Optical flow:

\[ f_x dx + f_y dy + f_t dt = 0 \]
\[ f_x u + f_y v + f_t = 0 \]

[2] RELATED WORK

Optical Flow estimation is vast area and survey of the entire area is beyond the scope of this paper. In this related work section, my goals are: 1) to present main components of optical flow methods. 2) to focus primarily on two main basic methods and coarse to fine algorithm.

Optical flow estimation has come far since the pioneering works of Lucas/Kanade [5] and Horn/Schunck [4]. Modern optical flow algorithms are reaching a point where they become suitable for deployment in real-world vision systems, e.g. [6]. Still, most state-of-the-art
methods continue to be variants of the continuous energy minimization framework of [4], i.e. they formulate an energy that aggregates data and smoothness costs over the image as a function of the flow field, and then seek to minimize it.

Optical flow is a commonly used method to estimate velocity distributions in a scene in image sequence processing. Many gradient-based methods such as the Horn–Schunck method [4] and Lucas–Kanade method [5] (developed during early studies conducted in the 1980s) have been developed to estimate the optical flow by calculating the brightness gradients of images [7], [8]. In most of these gradient based methods, partial derivatives with respect to temporal coordinates are calculated as brightness gradients. The coarse-to-fine framework was first proposed in [10, 11]. It has since been adopted by most optical flow algorithms to handle large displacement motions [9]. It was probably the first to note that the standard coarse-to-fine framework may not be sufficient. They proposed to modify the standard framework by using a linear scale-space focusing strategy to avoid convergence to incorrect local minima.

[3] PROPOSED WORK

Gradient method cannot give complete solution for optical flow fields because of aperture problem i.e., normal component parallel to gradient direction can be determined but tangential component perpendicular to gradient direction remains unsolved. We have analyzed two different solutions to the aperture problem in this paper.

[3.1] HORN & SCHUNCK METHOD

First we calculate fx, fy, ft using derivative masks [3]:

\[
\begin{bmatrix}
-1 & 1 \\
-1 & 1
\end{bmatrix} \quad \begin{bmatrix}
-1 & -1 \\
1 & 1
\end{bmatrix} \quad \begin{bmatrix}
1 & 1 \\
-1 & -1
\end{bmatrix}
\]

1) Apply mask1 to both images; add the responses to get fx.
2) Apply mask2 to both images; add the responses to get fy.
3) Apply mask4 to image1, apply mask3 to image2, add the responses to get ft.
4) Repeat calculating the following values until converges.

\[
P = f_x u_{av} + f_y v_{av} + f_t
\]
\[
D = \lambda + f_x^2 + f_y^2
\]
\[
u = u_{av} - \frac{f_x P}{D}
\]
\[
v = v_{av} - \frac{f_y P}{D}
\]

In which, Xav is the average value of X’s four neighbors.
[3.2] LUCAS & KANADE METHOD

Lucas & Kanade’s methods uses small windows (in my code, I used 3 by 3 window), and least squares method [3].

\[
\min \sum_i (f_{xi}u + f_{yi}v + f_t)^2 \\
\sum (f_{xi}u + f_{yi}v + f_t)f_{xi} = 0 \\
\sum (f_{xi}u + f_{yi}v + f_t)f_{yi} = 0 \\
\sum f_{xi}^2u + \sum f_{yi}f_{yi}v = -\sum f_{xi}f_t \\
\sum f_{yi}f_{yi}u + \sum f_{yi}^2v = -\sum f_{yi}f_t
\]

\[
\begin{bmatrix}
\sum f_{xi}^2 & \sum f_{xi}f_{yi} \\
\sum f_{xi}f_{yi} & \sum f_{yi}^2
\end{bmatrix}
[u, v] = \begin{bmatrix}
-\sum f_{xi}f_t \\
-\sum f_{yi}f_t
\end{bmatrix}
\]

Because when implementing coarse-to-fine, I used Lucas & Kanade method.

[3.3] COARSE TO FINE METHOD

The small motion assumption, regular optical flow methods work badly if the object we are tracking moves a long distance, under this circumstance, coarse-to-fine method helps a lot.

What coarse-to-fine is simply building image pyramids for each images, and by doing optical flow on each layer of pyramid, to get rid of the small motion constraint. For example, if we get a 3-layer pyramid, then one pixel in the top layer can represent 4 pixels of distance in the lowest layer. The following [Figure-2] shows how this method works:

![Figure 2. Pyramid during coarse to fine](image-url)
Combining ingredients of local and global method for estimation:

Energy = $\int \phi$ (Data) + $\int \phi$ (“Smoothness”)

Where,

$\Phi$ = Combined using robust statistics and computed coarse to fine

[4] MATHEMATICAL MODEL

[4.1] EARLY LINEARIZATION

Here we are going to see Coarse-to-fine mathematical model for calculating optical flow

Early linearization (i.e. no warping). We use quadratic smoothness term (i.e. Tikhonov regularizer).

Here S is our system,

$S = \{I, O, DD, NDD\}$

Where,

$I = \{I_0, I_1, scl, sclFactor | Input of the model\}$

$O = \{u, v | output of the model\}$

$D = \{I_0, I_1 | Deterministic data\}$

$NDD = \{u, v | Non deterministic data\}$

1) Set $u = 0, v = 0$.
2) Create image pyramid
   $[I_{scl0 \{}{}, I_{scl1 \{}{]}]=pyramid (I_0, I_1, scl, sclFactor)$
3) Approximate derivatives for $I_0$ and $I_1$:
4) Solve for new $u$ and $v$

$f \{u,v\} \rightarrow f \{SOLVER (u, v, nLoops, )\}$

5) $f(\boldsymbol{uv}) = \begin{cases} Prolongate(\boldsymbol{u, v, sclFactor}), & s - 1 > 0 \\ No \ prolongate, & otherwise \end{cases}$

[4.2] LATE LINEARIZATION

Here we are going to see Coarse-to-fine mathematical model for calculating optical flow

Late linearization (i.e. uses warping). We use robust error functions in both the data and the smoothness terms.

$S = \{I, O, DD, NDD\}$

Where,
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\( I = \{ I_0, I_1, \text{scl, sclFactor} \mid \text{Input of the model} \} \)

\( O = \{ u, v \mid \text{output of the model} \} \)

\( D = \{ I_0, I_1 \mid \text{Deterministic data} \} \)

\( \text{NDD} = \{ u, v \mid \text{Non deterministic data} \} \)

1) Create image pyramid

\[
[\text{Iscl0} \ {\{ \} \} \text{Iscl1} {\{ \}]} = \text{pyramid}(I_0, I_1, \text{scl, sclFactor})
\]

2) Warp image as per \( u \) and \( v \)

\( I_{w,0} = \text{warp}(I_0,0,u,v) \)

3) Calculate penalizer function values for data

\[
\Psi' ((E_{k,D})) = \frac{\delta I_k}{\delta t} - \frac{\delta I_{w,0}}{\delta x} \delta u - \frac{\delta I_{w,0}}{\delta y} \delta v
\]

4) Calculate diffusion weights and solve for new \( du, dv \).

5) Update \( u, v \)

\( u = u + du \)

\( v = v + dv \)

6) Interpolate (prolongate) solution

\[
f(uv) = \begin{cases} 
\text{Prolongate}(u,v,\text{sclFactor}), & s - 1 > 0 \\
\text{No prolongate}, & \text{otherwise}
\end{cases}
\]

[5] RESULTS

Input Image

Following [Figure-3] is the snapshot of my input image.

![Figure 3. Car Test Result](image-url)
Output Images

1. Following [Figure-4] is the output of the Horn & Schunck result.

![Figure 4. Horn & Schunck result](image)

2. Following [Figure-5] is the output of the Lucas & Kanade result.

![Figure 5. Lucas & Kanade result](image)

3. Following [Figure-6] is the output of the Lucas & Kanade with coarse to fine which is better than other results.
[6] CONCLUSION

The determination of optical flow forms an important low level stage for motion analysis in computer vision. However, as we have seen, optical flow fields are not trivial to determine. The aperture problem fundamentally limits reliability in many areas by restricting the calculated optical flow to the component along the image gradient. Many methods try to solve this problem and find the entire flow vector. Combining local and global methods can be used to yield much better results to long range displacement. Also the coarse to fine methods gives us the result which is better compared to others.
REFERENCES


